INTEGRABILITY, CONFORMAL BOOTSTRAP AND DEFECTS IN N=4 SYM

HeI kick-off meeting, January 2025

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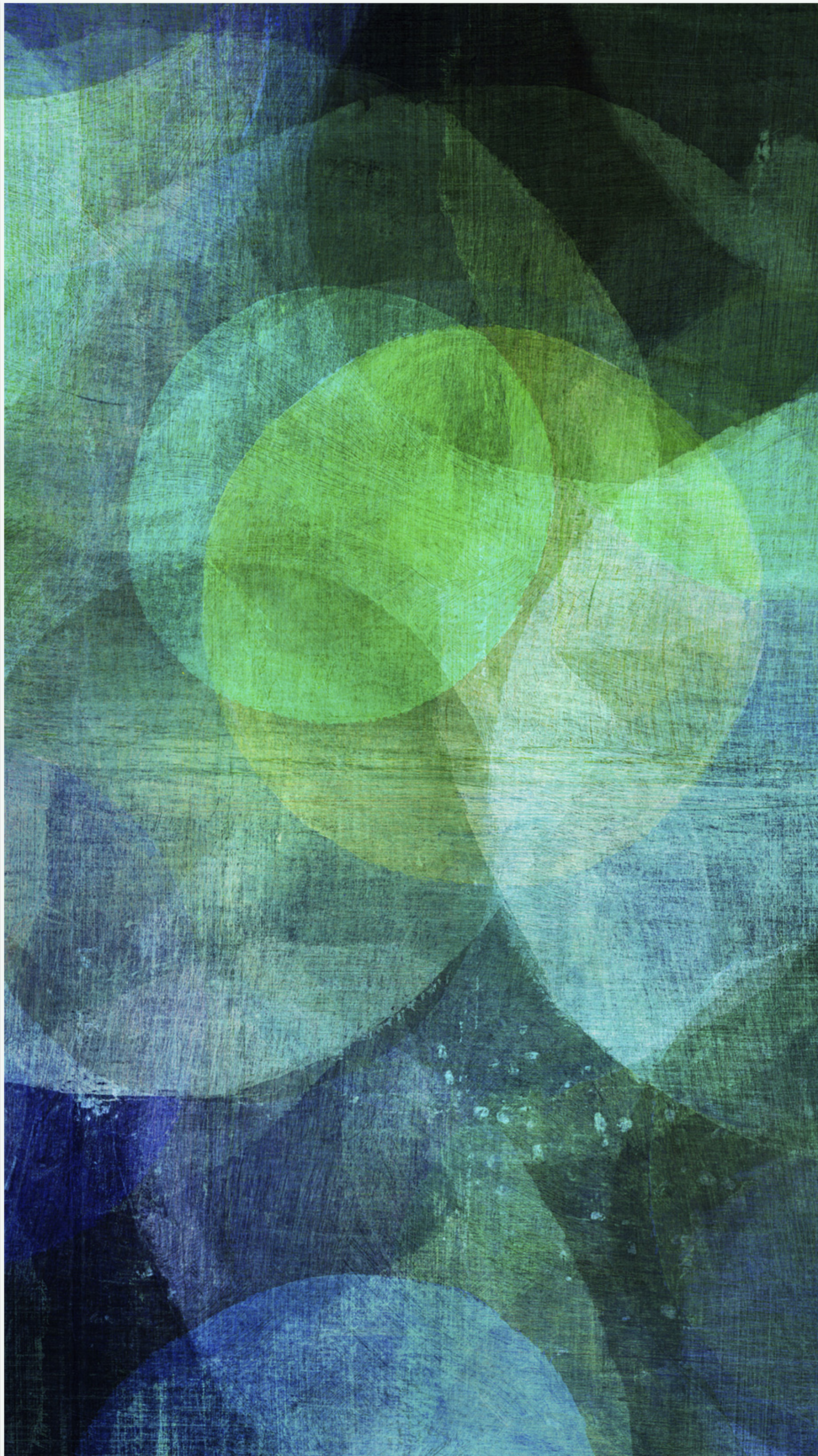
WP1-Integrability and Bootstrability

UNIVERSITÀ DI TORINO

works with N. Gromov, J. Julius, M. Preti, N. Sokolova

arXiv: 2107.08510, 2203.09556, 2211.03203, 2312.11604, 2412.07624

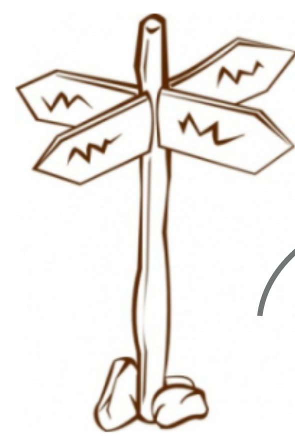


N=4 SYM is often considered a “modern hydrogen atom”

It can be interesting if you come from different directions…

Conformal Field Theory (CFT)

QCD



N=4 SYM

AdS/CFT

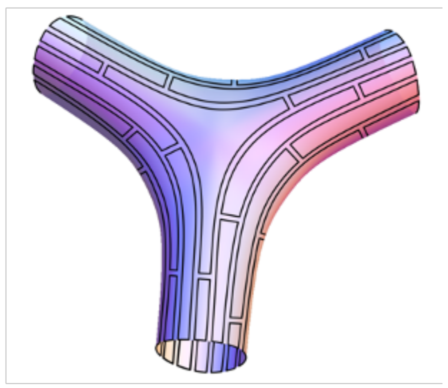
String Theory on Curved Space

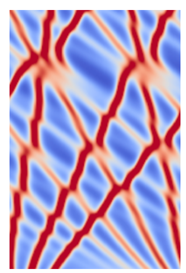
two non-perturbative and very different approaches

Integrability:

Large N but finite coupling

It's **magic**: miracles in auxiliary 2d world





Solve **one theory**

Exact analytical results
Not yet understood for all observables

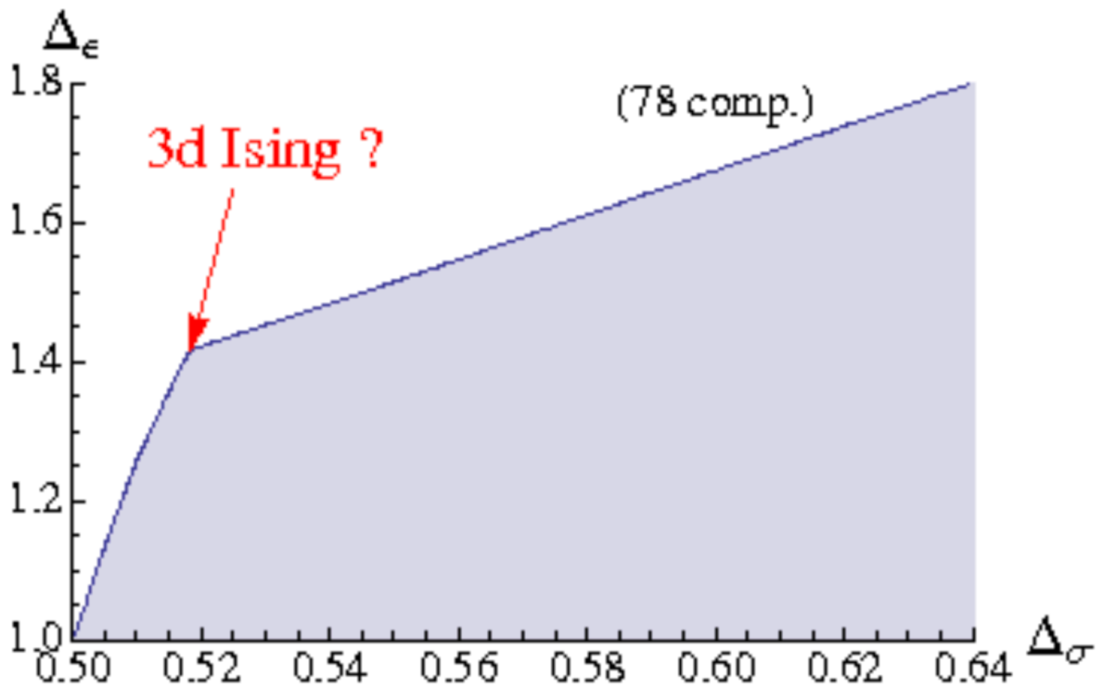
Conformal Bootstrap

Finite N and finite coupling

Exploits theory-independent principles: **OPE**, **locality**, **unitarity, Conformal symmetry**

Constrain **space of theories**

At finite coupling usually gives rigorous bounds on observables



Bootstrap gives allowed regions

special theories live close to the boundaries

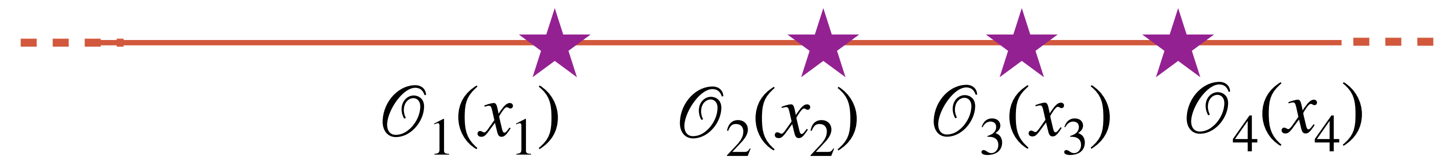
N=4 SYM seems to be hidden deeper

To constrain it, combine the two methods: **Bootstrability**

Conformal Bootstrap + data from Integrability

Nice observables to test this idea

excitations of (supersymmetric)
straight Wilson lines



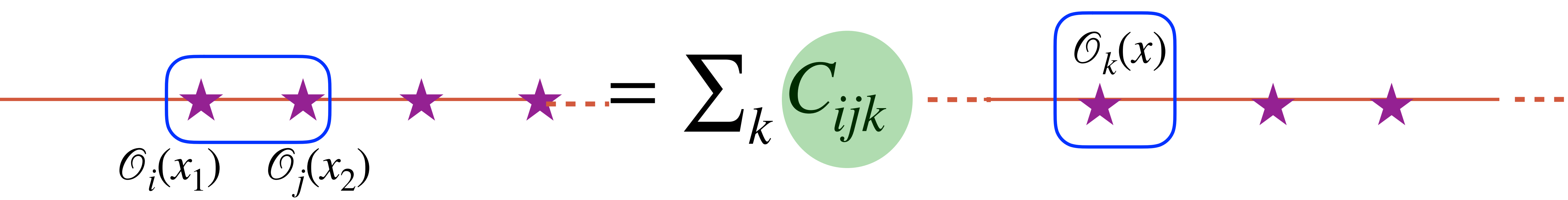
1d CFT axioms:

1)

$$x \to x' = \frac{ax+b}{cx+d}$$

$$\langle \mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\rangle = \langle \mathcal{O}_1(x_1')\ldots\mathcal{O}_n(x_n')\rangle \prod_{i=1}^{n} \left|\frac{\partial x_i'}{\partial x_i}\right|^{\Delta_i}$$

2) Associative Operator
Product Expansion

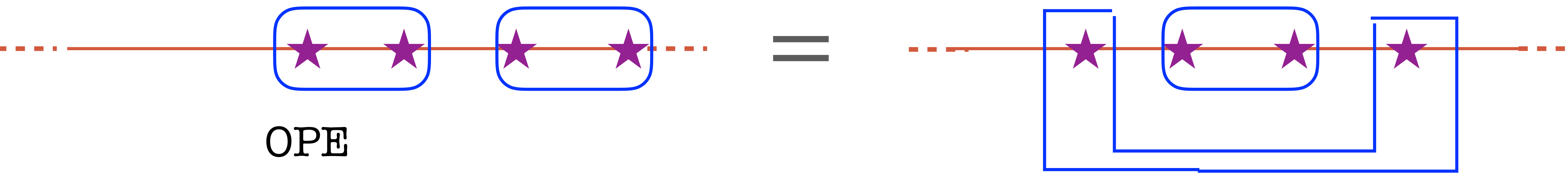


CFT data:

Δ_i C_{ijk}

Spectrum OPE coefficients

Will start by considering 4pt of identical lightest operators of dimension $\Delta_{ext} = 1$



Crossing equations:

$$\sum_{\Delta_n} C_n^2 \mathcal{G}_{\Delta_n}(x) = \mathcal{F}(x, g) \qquad \forall x = \frac{x_{12}x_{34}}{x_{14}x_{23}}$$

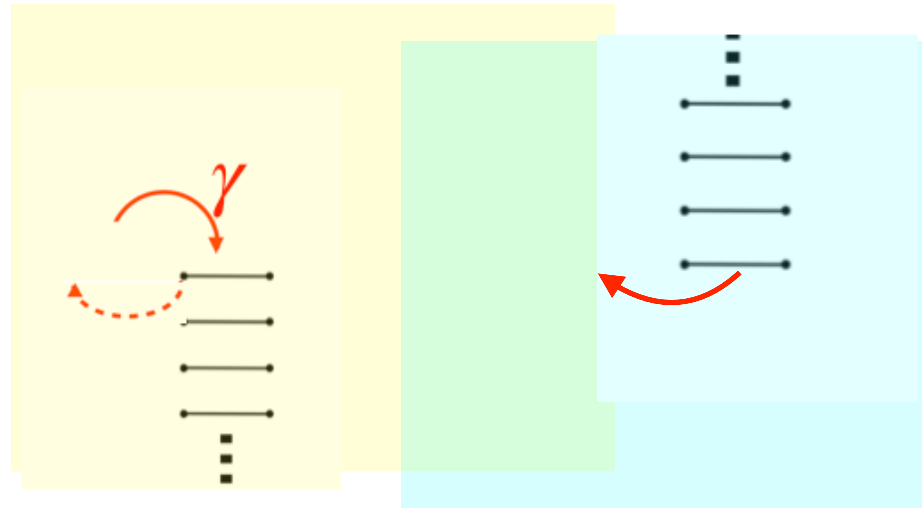
$\mathcal{G}_{\Delta}(x)$, $\mathcal{F}(x, g)$: explicit

Integrability

A key result: the **Quantum Spectral Curve**

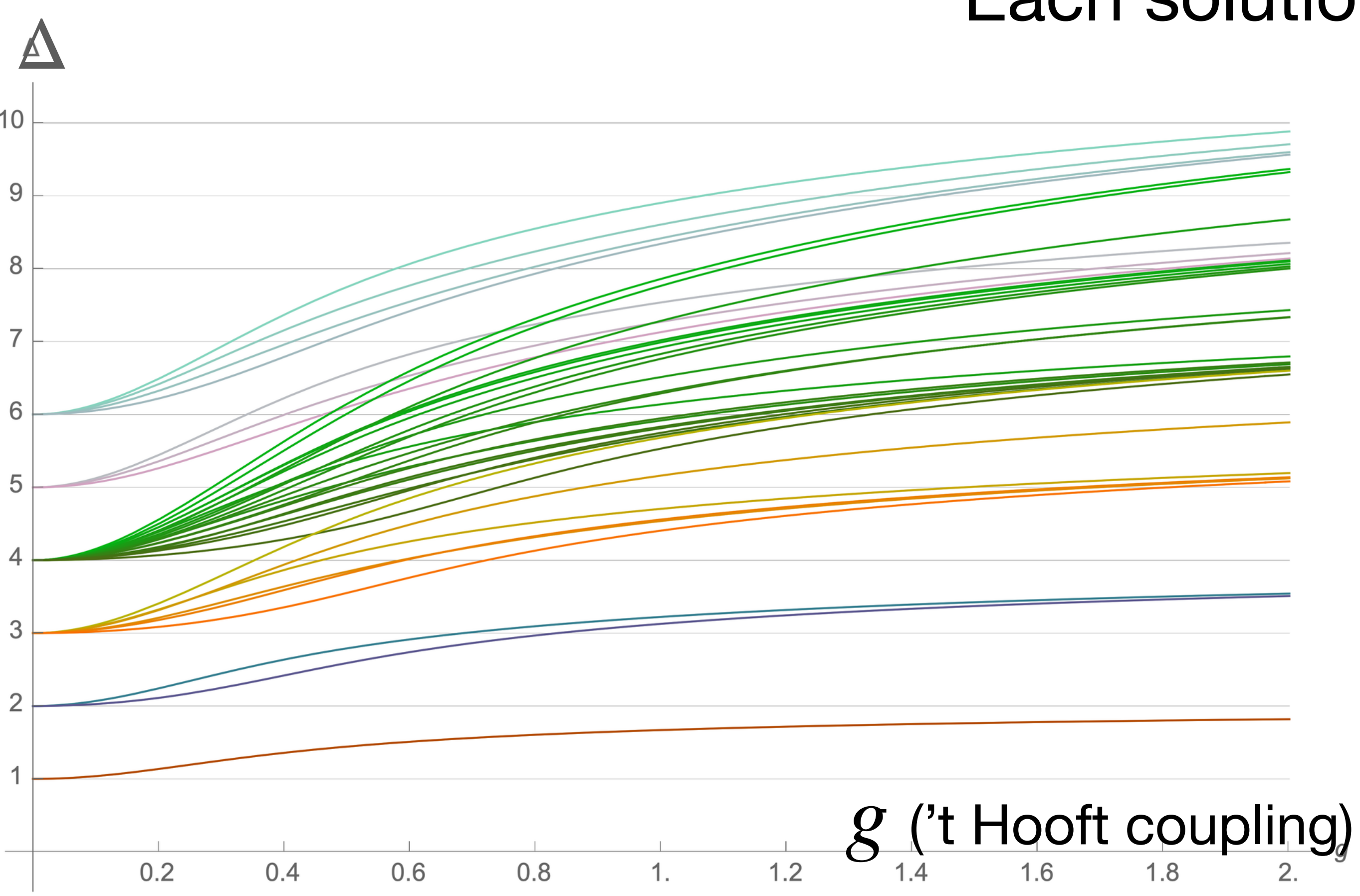
[Gromov,Kazakov, Leurent Volin] + …

= equations for the "Q-functions"



$$\mathbf{Q}(u) \sim u^{\Delta}, \;\; u \to \infty$$

Each solution = one operator



Can compute spectrum (although only state-by-state)

Integrability

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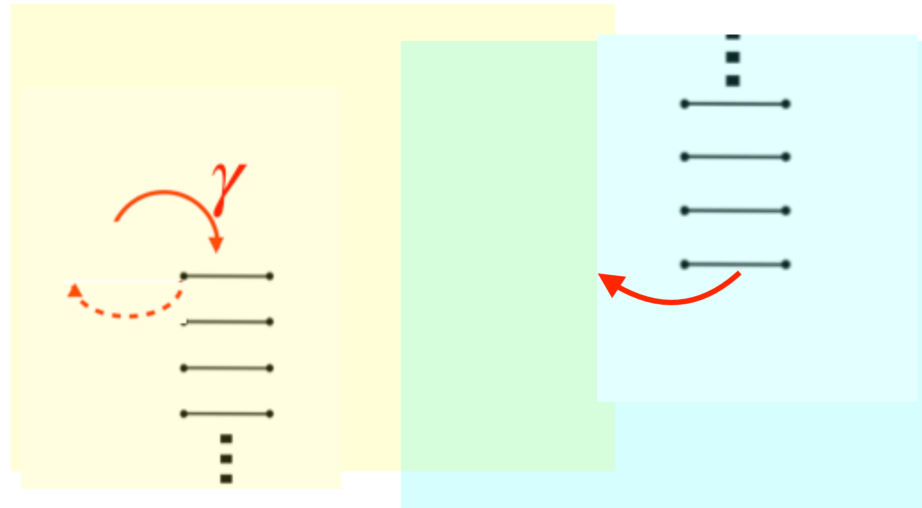

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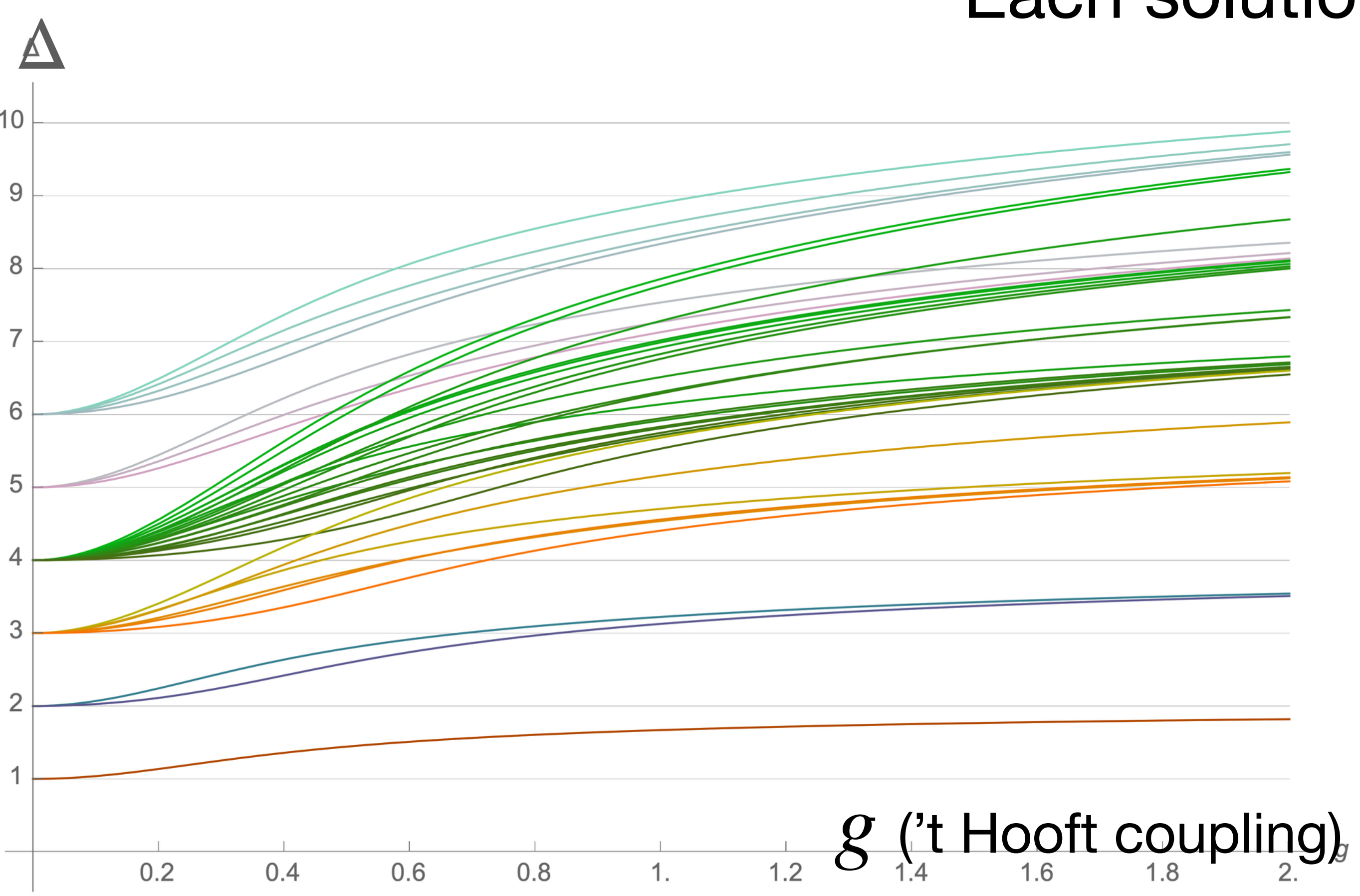


Also expected:

Correlation functions are
"overlaps of Q-functions"

$$C_{123} \sim \int\int \dots \int \prod_i \mathbf{Q}_1(u_i)\mathbf{Q}_2(u_i)\mathbf{Q}_3(u_i)\mu_{\mathbf{3}}(u_1, u_2, \dots)$$

Not fully understood yet

Can compute spectrum (although only state-by-state)

Bootstrap

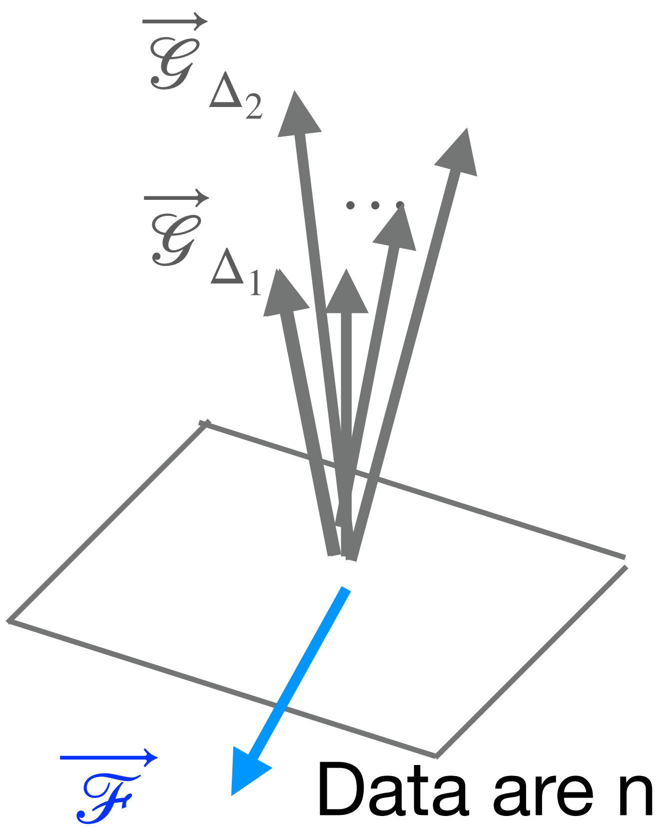
$$\sum_{\Delta} C_{\Delta}^2 \, \mathscr{G}_{\Delta}(x) = \mathscr{F}(x, g)$$

[El-Showk,Paulos,Poland,Rychkov,
Simmons-Duffin,Vichi '12] + …

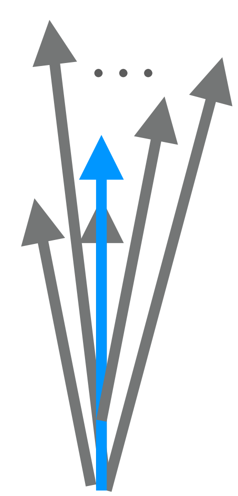
differentiate in x

$$\sum_{\Delta} C_{\Delta}^2 \, \overrightarrow{\mathscr{G}}_{\Delta} = \overrightarrow{\mathscr{F}}, \qquad C_{\Delta}^2 \geq 0$$

Can check for consistency of proposed data.
E.g. we can check some hypothesis on the spectrum…



Data are not consistent



Consistent

can treat all
Δ_i above a cutoff as unknowns

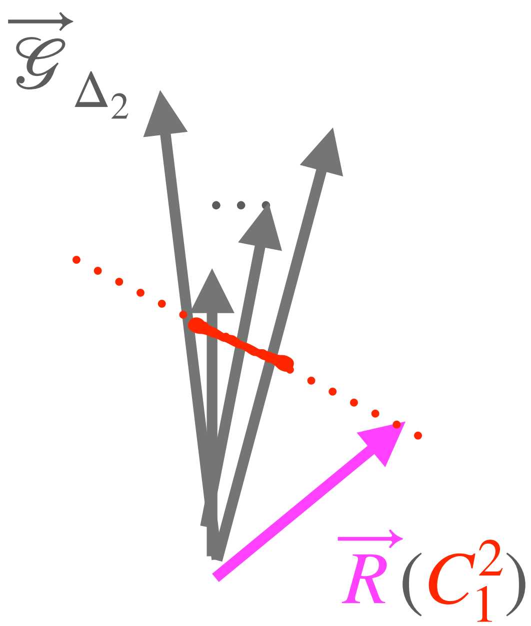
(separating hyperplane)

$$\sum_{\Delta} C_{\Delta}^{2}\, \mathscr{G}_{\Delta}(x) = \mathscr{F}(x, g)$$

differentiate in x

$$\sum_{\Delta} C_{\Delta}^{2}\, \overrightarrow{\mathscr{G}}_{\Delta} = \overrightarrow{\mathscr{F}}, \qquad C_{\Delta}^{2} \geq 0$$

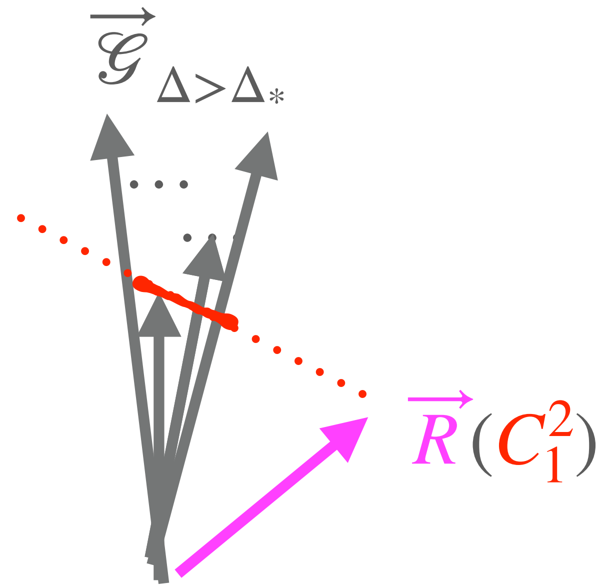
Can check for consistency of proposed data.
... or e.g. bound some OPE coefficients



$$\sum_{n\geq 2} C_{\Delta_n}^{2}\, \overrightarrow{\mathscr{G}}_{\Delta_n} = \overrightarrow{\mathscr{F}} - C_1^2 \overrightarrow{\mathscr{G}}_{\Delta_1} \equiv \overrightarrow{R}(C_1^2),$$

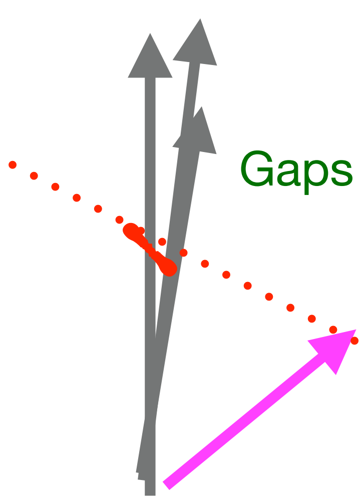
Allowed range for C_1^2

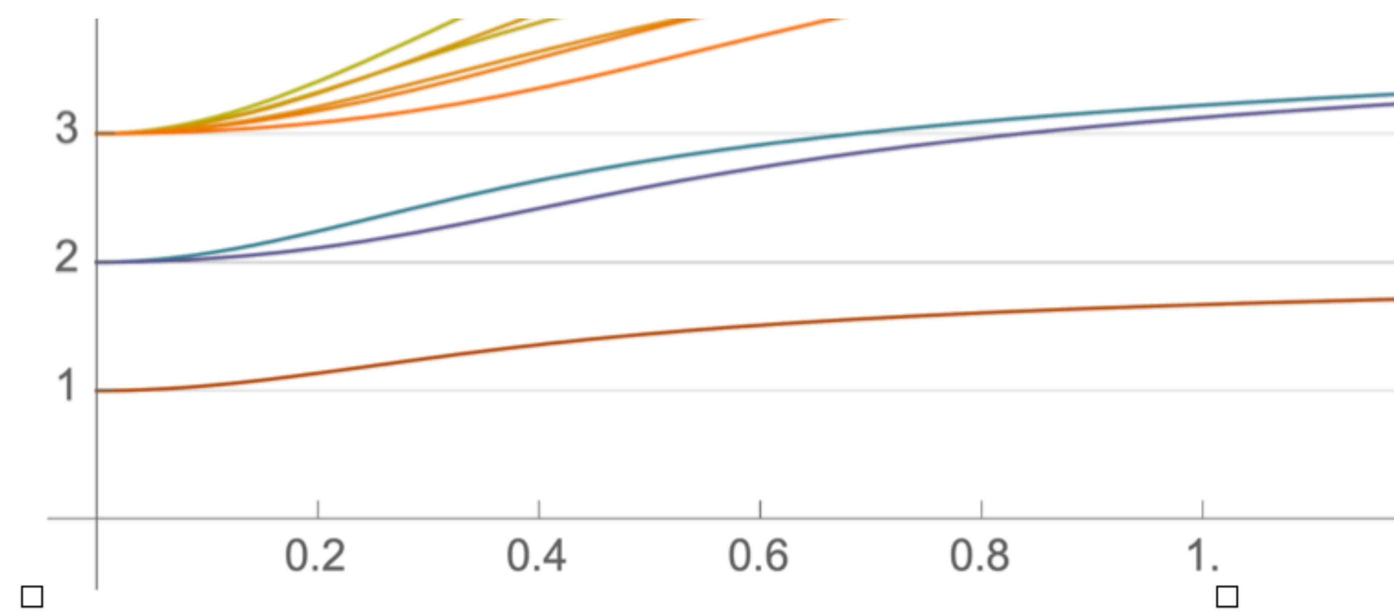
How do integrability spectral data help?



If we know little of the spectrum

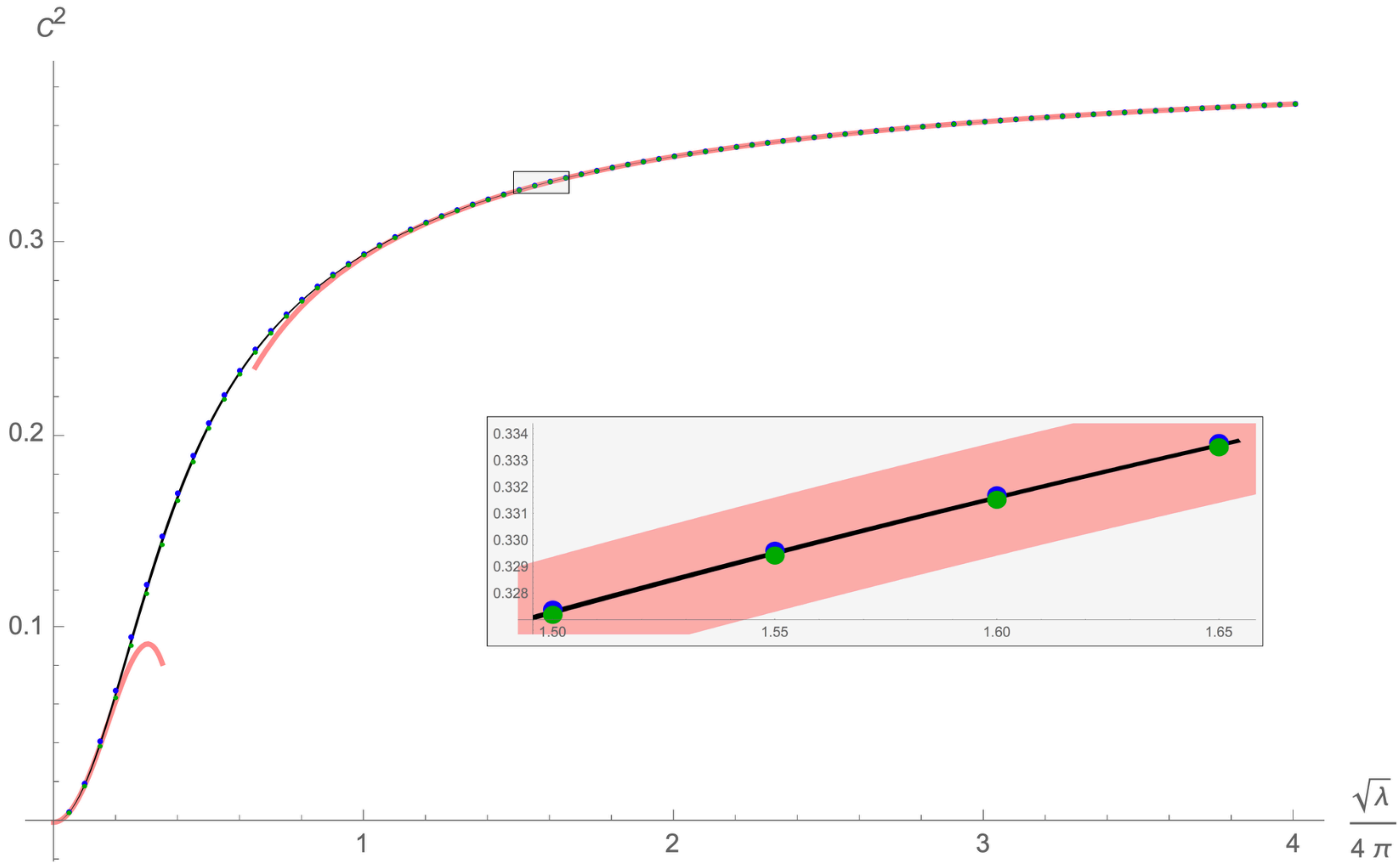
If we know where some **gaps** in Δ are…





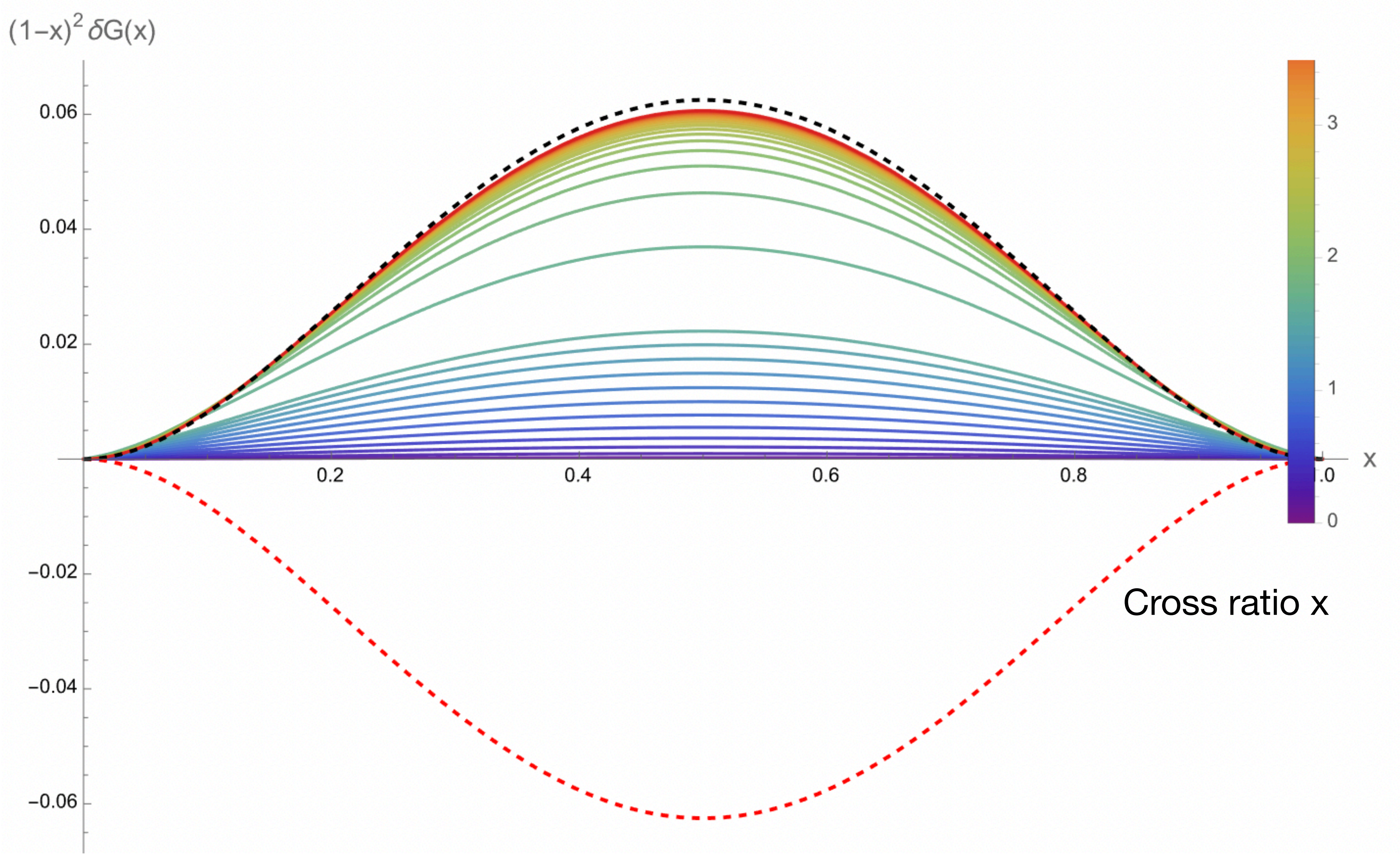
Narrower bounds for OPE coff.

Examples of results: the leading OPE coefficient in the line CFT

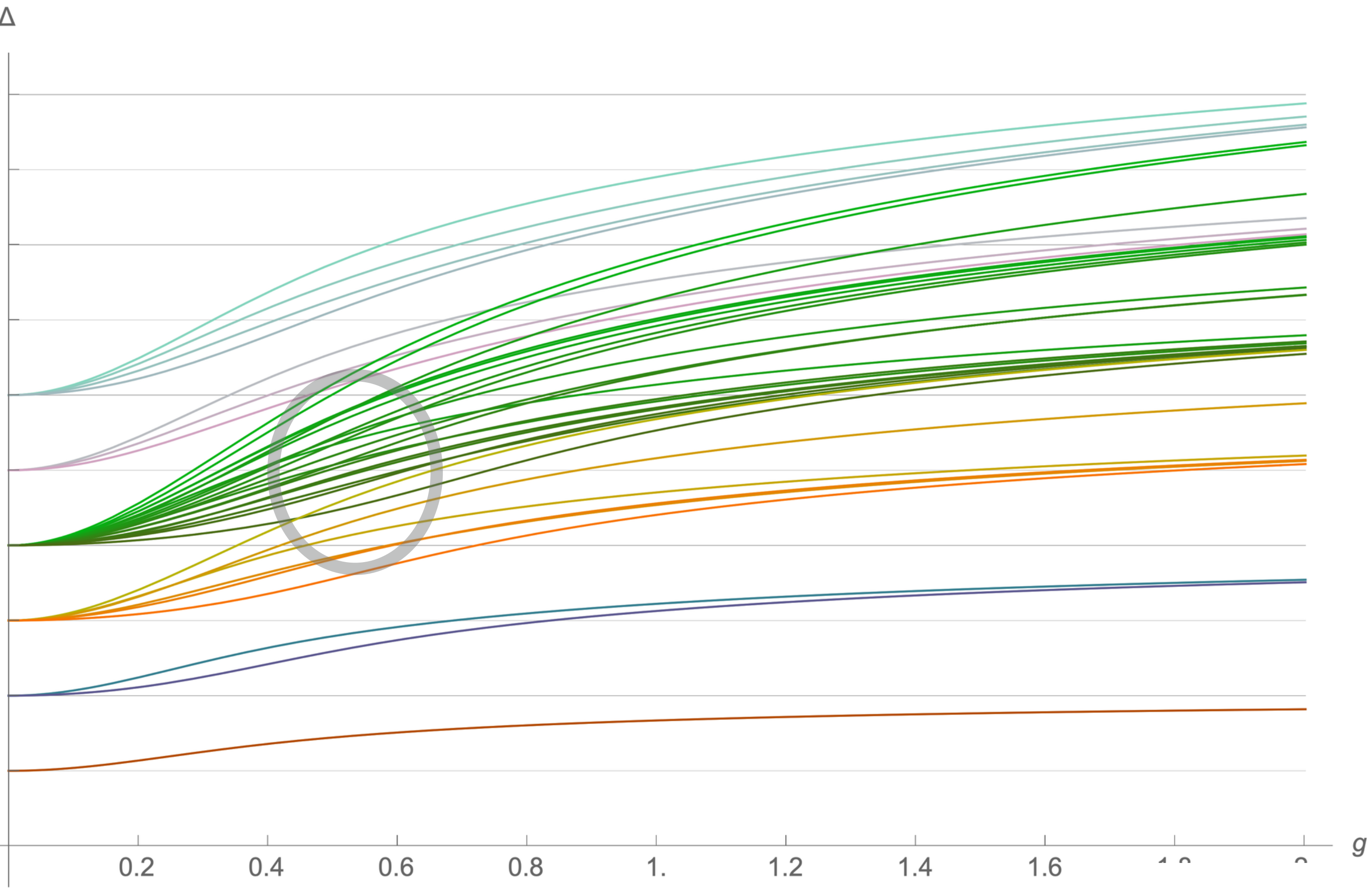


('t Hooft coupling)

The simplest 4-point function itself



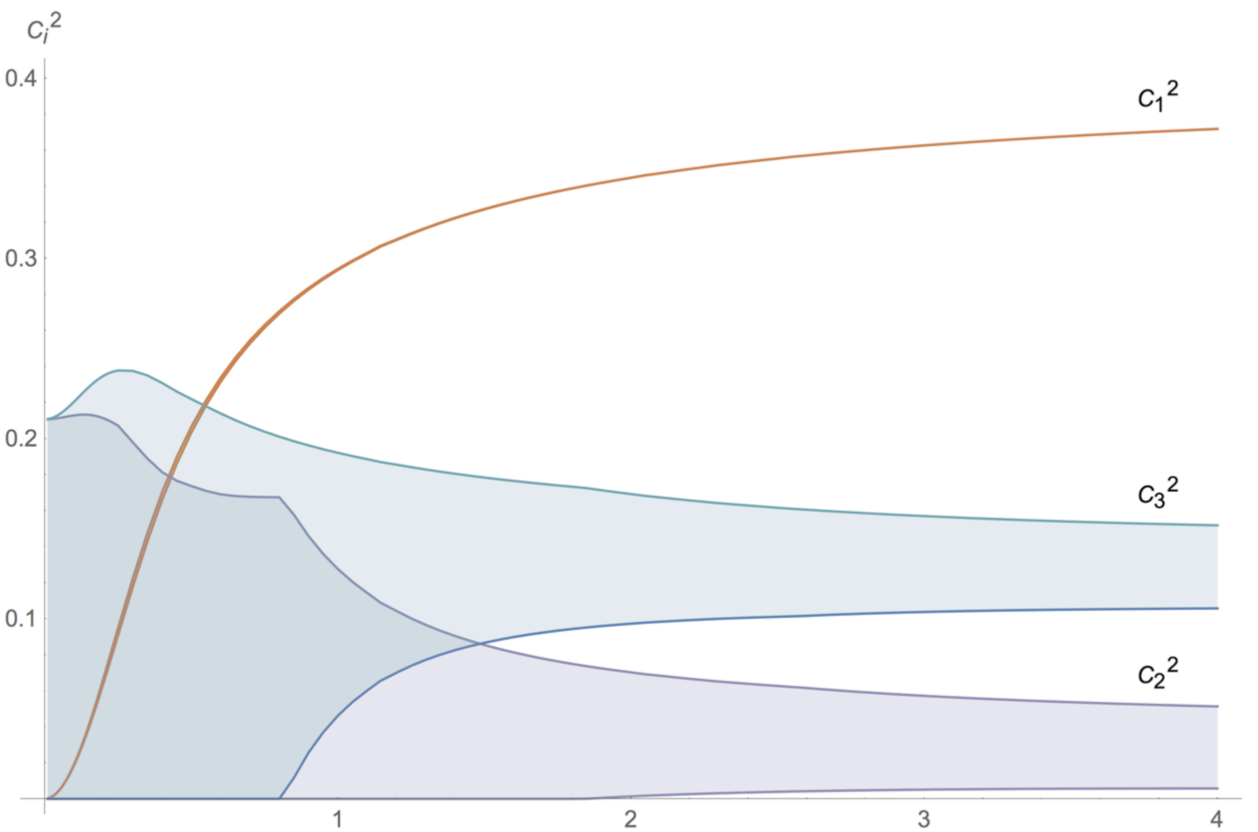
Challenges of this approach



Higher part of the spectrum has very narrow gaps at finite coupling

… its knowledge is much less impactful

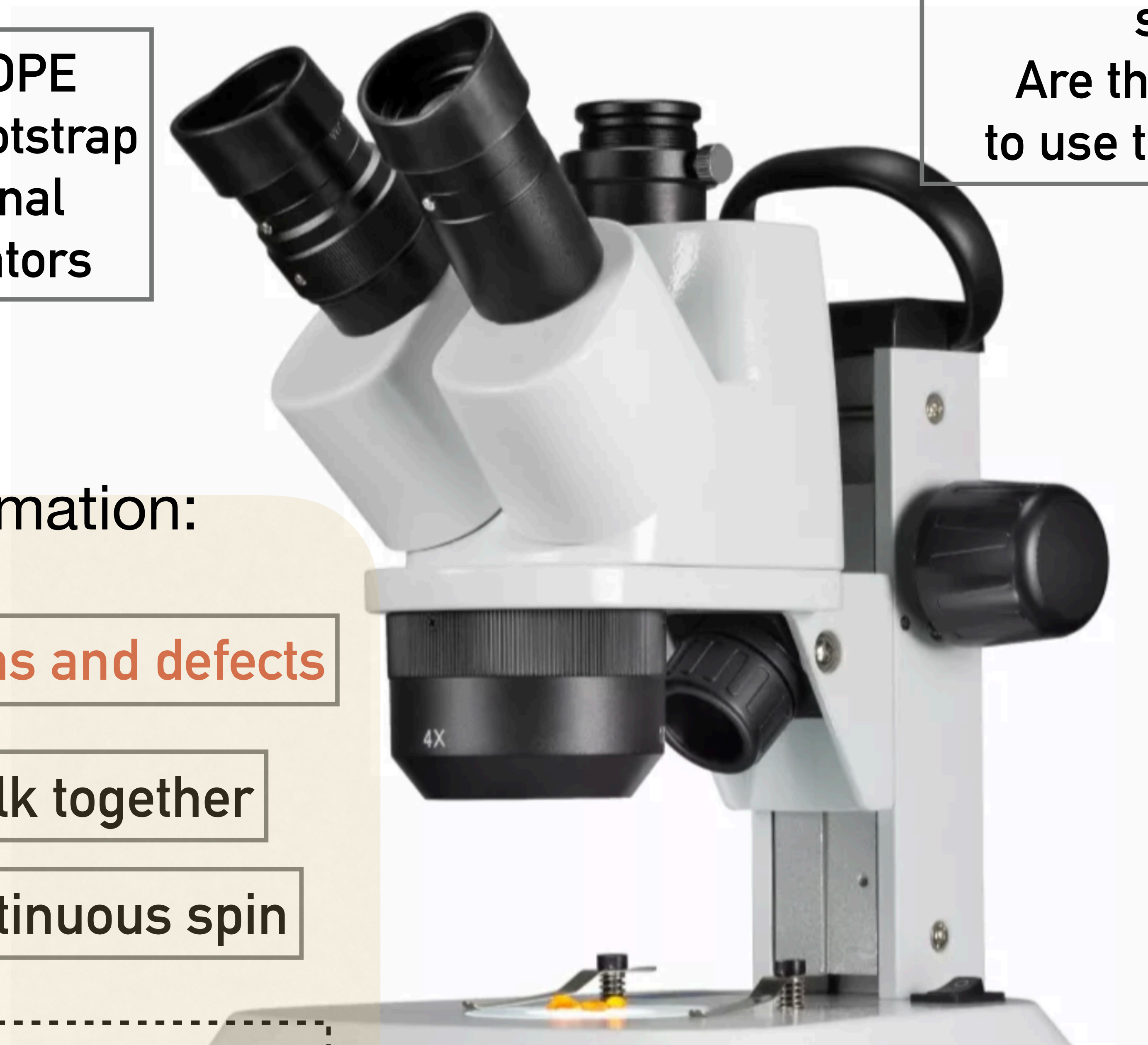
We also find much less sharp results for higher OPE coefficients



What can we do?

Integrability also describes higher part of the spectrum.
Are there other ways to use this information?

To focus on higher OPE coefficients: mixed bootstrap systems with external non-protected operators



introduce new information:

The role of deformations and defects

Defects and bulk together

Spectrum at continuous spin

$\ldots C_{123}$ from integrability?

Getting information from what lies outside the Wilson line

$$\mathcal{W}_{\mathcal{C}} = \frac{1}{N}\,\mathrm{Tr}\,\mathrm{P}\exp\int_{\mathcal{C}} dt\;\left(i\,A_\mu \dot{x}(t)^\mu + |\dot{x}(t)|\;n\cdot\Phi\right)$$

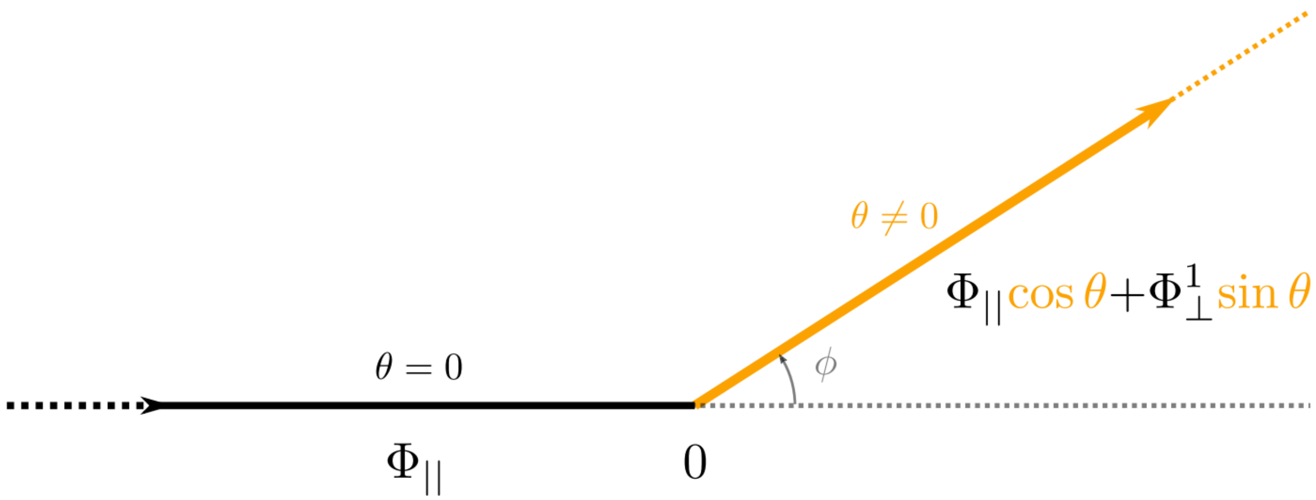
one of six scalar fields of N=4 SYM

$$\overrightarrow{n}\cdot\overrightarrow{\Phi} \equiv \Phi_{||} \equiv \Phi_6$$

parallel

$$\left\{\Phi^i\right\}_{i=1}^{5} \equiv \left\{\Phi_\perp^i\right\}_{i=1}^{5}$$

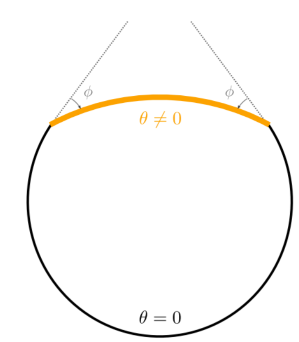
orthogonal



Forming a cusp

Freedom to vary angle remaining integrable

Kind of 2-pt function



$$\langle\, \mathcal{W}_{\mathrm{cusp}}\,\rangle \sim (\epsilon_{\mathrm{UV}})^{\Gamma_{\mathrm{cusp}}(g,\phi,\theta)}$$

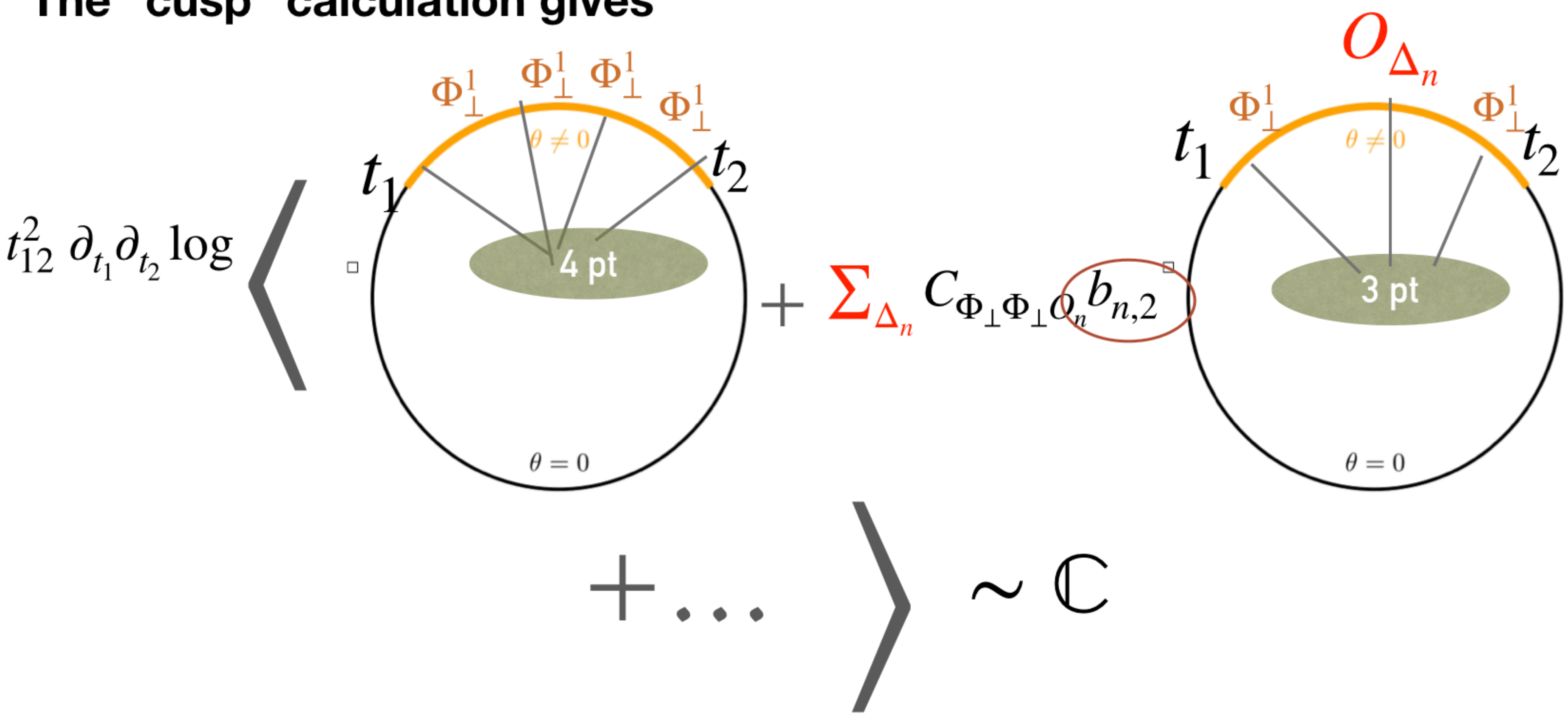
➤ $\Gamma_{\textbf{cusp}}$ is also known from integrability!!

[Drukker '12] [Correa, Maldacena, Sever '12]
[Gromov, Levkovich-Maslyuk '16]

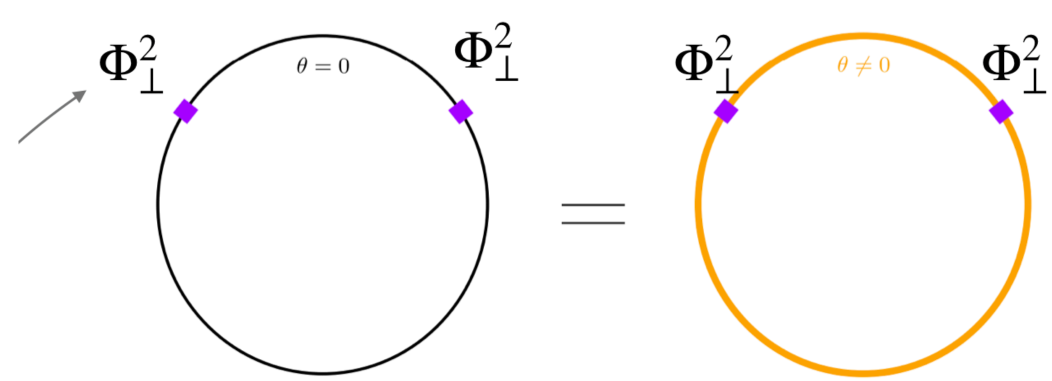
$$\mathcal{A}_{\rm CFT}(\theta) \sim \mathcal{A}_{\rm CFT}(0) + \delta\mathcal{A}_{\rm CFT}$$

$$\delta\mathcal{A}_{\rm CFT} = \mathbf{s}\int dt\, O_{\Phi_\perp^1}(t) + \sum_{k=2}^{\infty} \mathbf{s}^k \sum_n b_{n,k}\,\epsilon^{\Delta_n-1}\int dt\, O_n(t).$$

The “cusp” calculation gives

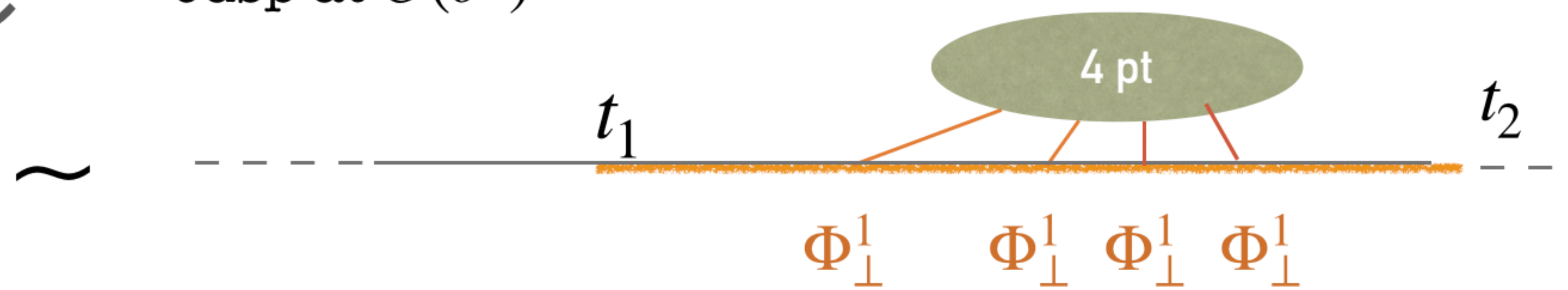


Key constraint:



- One way to get constraints on the original 1d CFT is to expand at small deformation

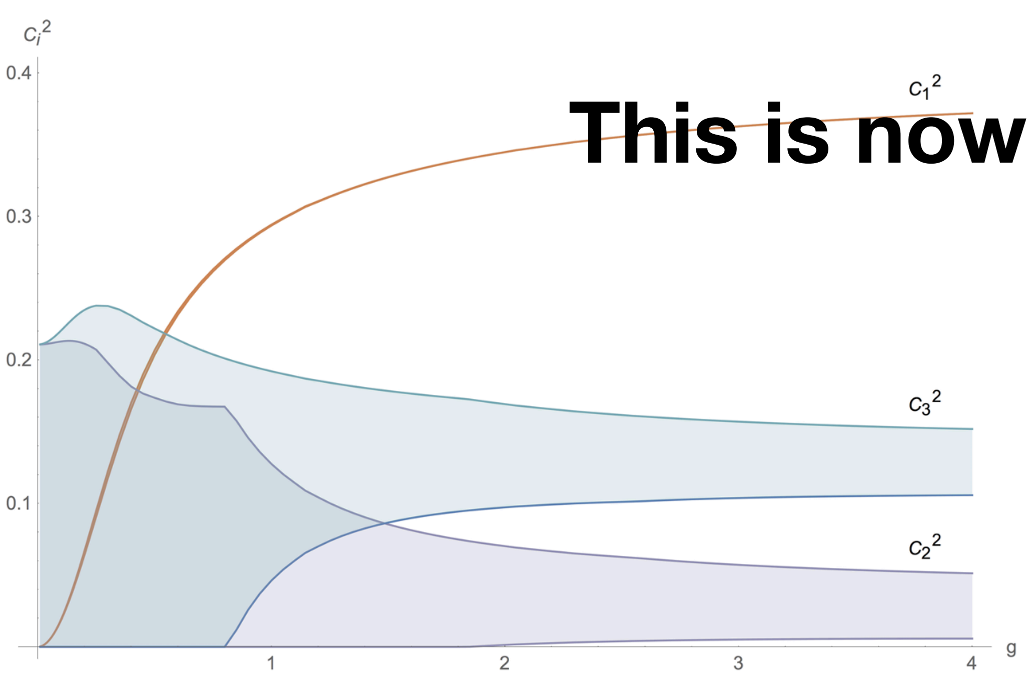
$\mathbb{C}$ $\sim$ cusp at $O(\theta^4)$



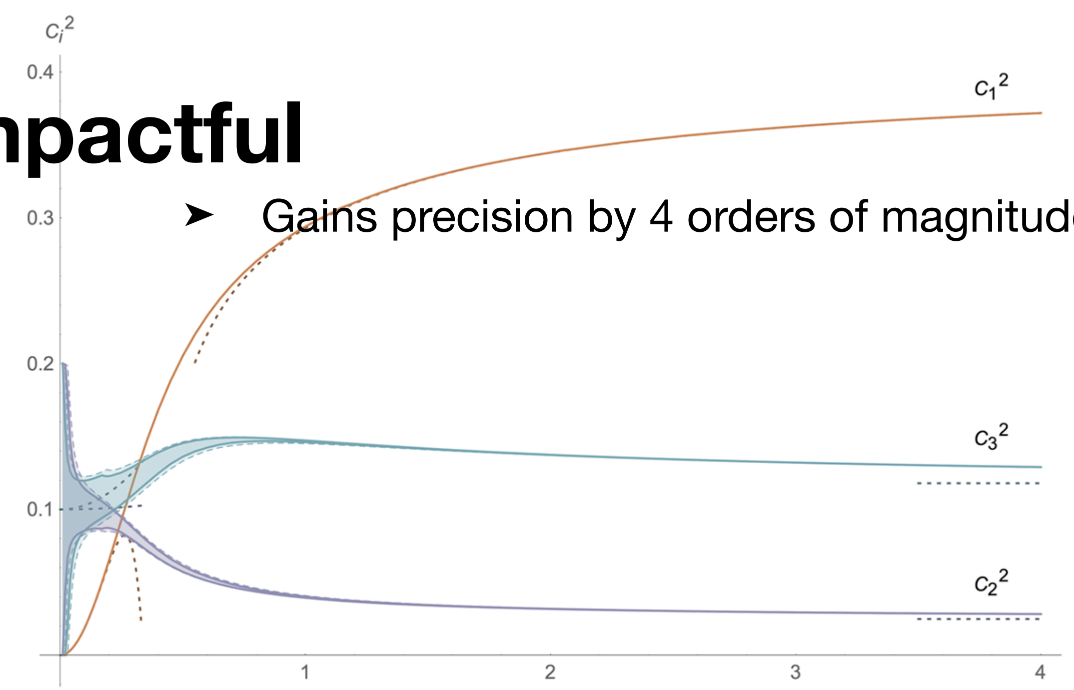
- Two integrated correlator identities-> new sum rules

[AC J. Julius, N. Gromov, M. Preti '22]
[N. Drukker, Z. Kong, G. Sakkas '22]

$$\sum_\Delta C_\Delta^2\, b_{i,\Delta} = r_i, \quad i = 1{,}2$$



This is now very very impactful

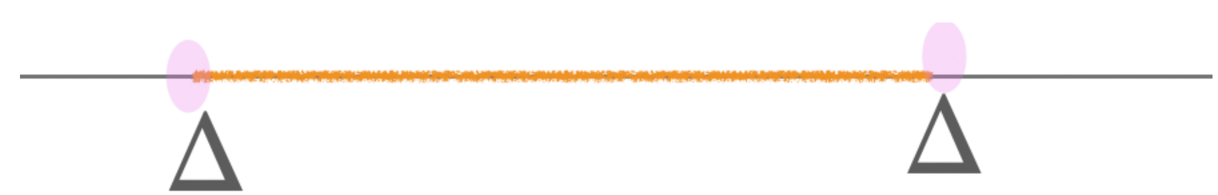


- Gains precision by 4 orders of magnitude!

These were just two of the simplest integrable deformations.
There should be many more such identities.

Integrated n-point functions … (cf. multi-point bootstrap)

Integrated non-BPS 4-pt functions…



Integrated local correlators from conformal deformations of the bulk theory…

- Combining all these methods seems very powerful

Conformal Bootstrap + Integrability & including all kinds of Defects

Good luck to our network!

Small bibliography

Bootstrability for bulk operators [Caron-Huot, Coronado, Zahraee 23] [Caron-Huot, Coronado, Zahraee 24]

Bootstrability with machine learning [Niarchos, Papageorgakis, Richmond, Stapleton Woolley 23]

Defects and integrated correlators [Billo Goncalves Lauria Meineri '16] [AC J. Julius, N. Gromov, M. Preti '22] [N. Drukker, Z. Kong, G. Sakkas '22] [Gabai Sever Zhong '25]

Integrability progress for 3-point functions [Basso Komatsu Vieira 15] [Basso Georgoudis Sueiro 22] [AC Gromov Levkovich-Maslyuk 18][Bercini Homrich Vieira 22]

Nice people in Torino
who work on topics very close to the goals of HeI (WP1 & WP2)

Lorenzo Bianchi

Marco Billo`

Marco Meineri

Roberto Tateo

Nicolo` Primi
Stefanos Kousvos
Nicolo` Brizio
Elia De Sabbata
Thekla Lepper
Andrea Mattiello
David Abner Gutierrez Romero

Mariaelena Boglione

Emanuele Roberto Nocera

Andrea Signori

José Osvaldo Gonzalez Hernandez
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